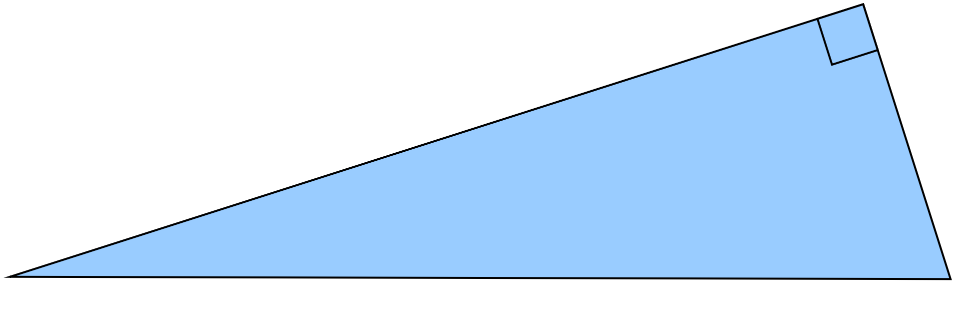
Geometric Mean in right triangles

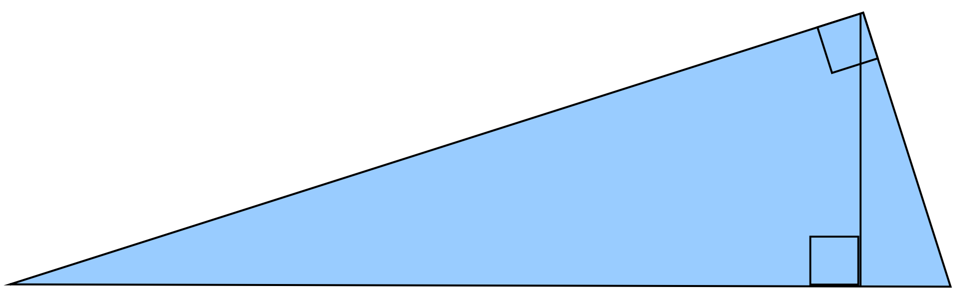
Geometric Mean

- We start with a Right Triangle



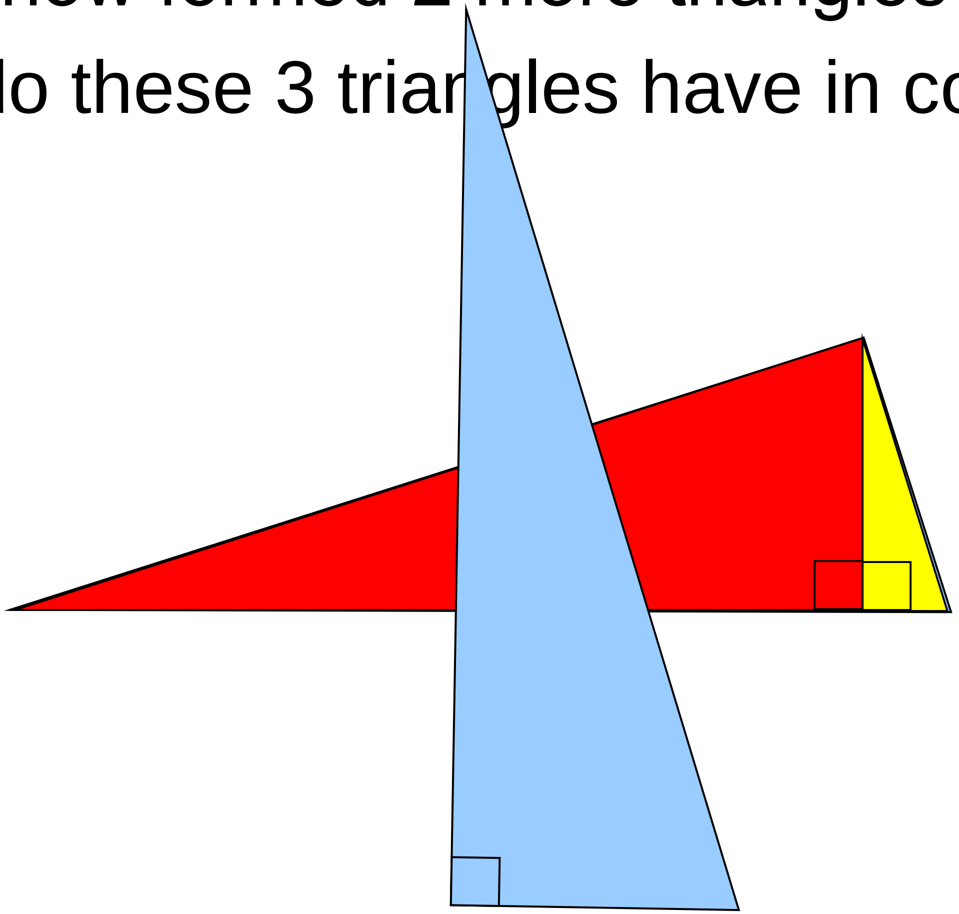
Geometric Mean

- Let's draw its altitude.



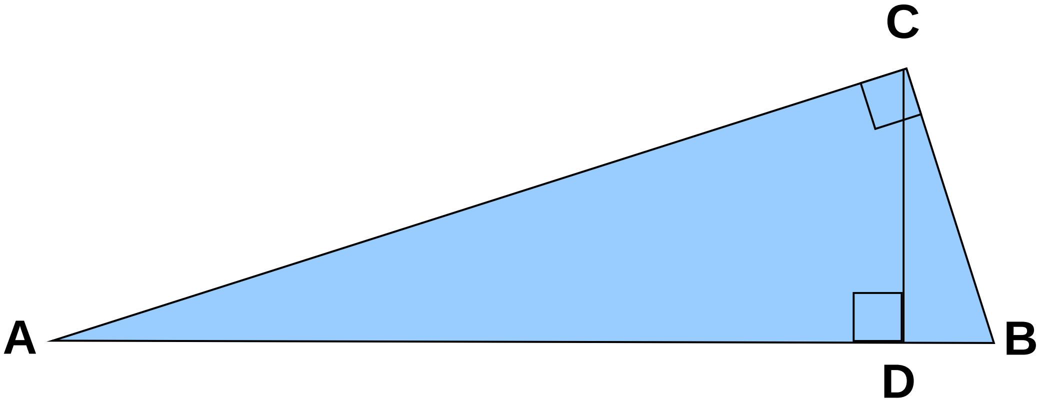
Geometric Mean

- We've now formed 2 more triangles – 3 in all!
- What do these 3 triangles have in common?



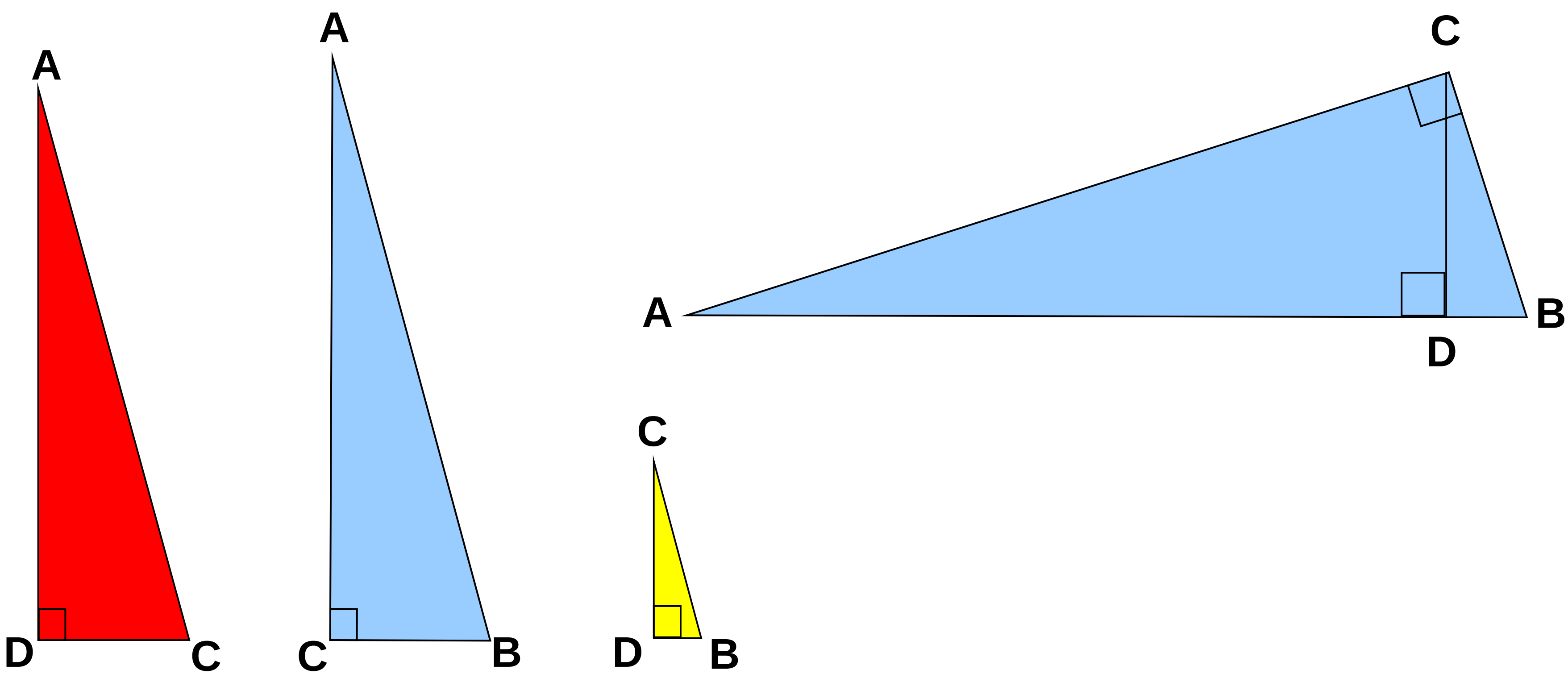
Geometric Mean

- Let's consider the original diagram



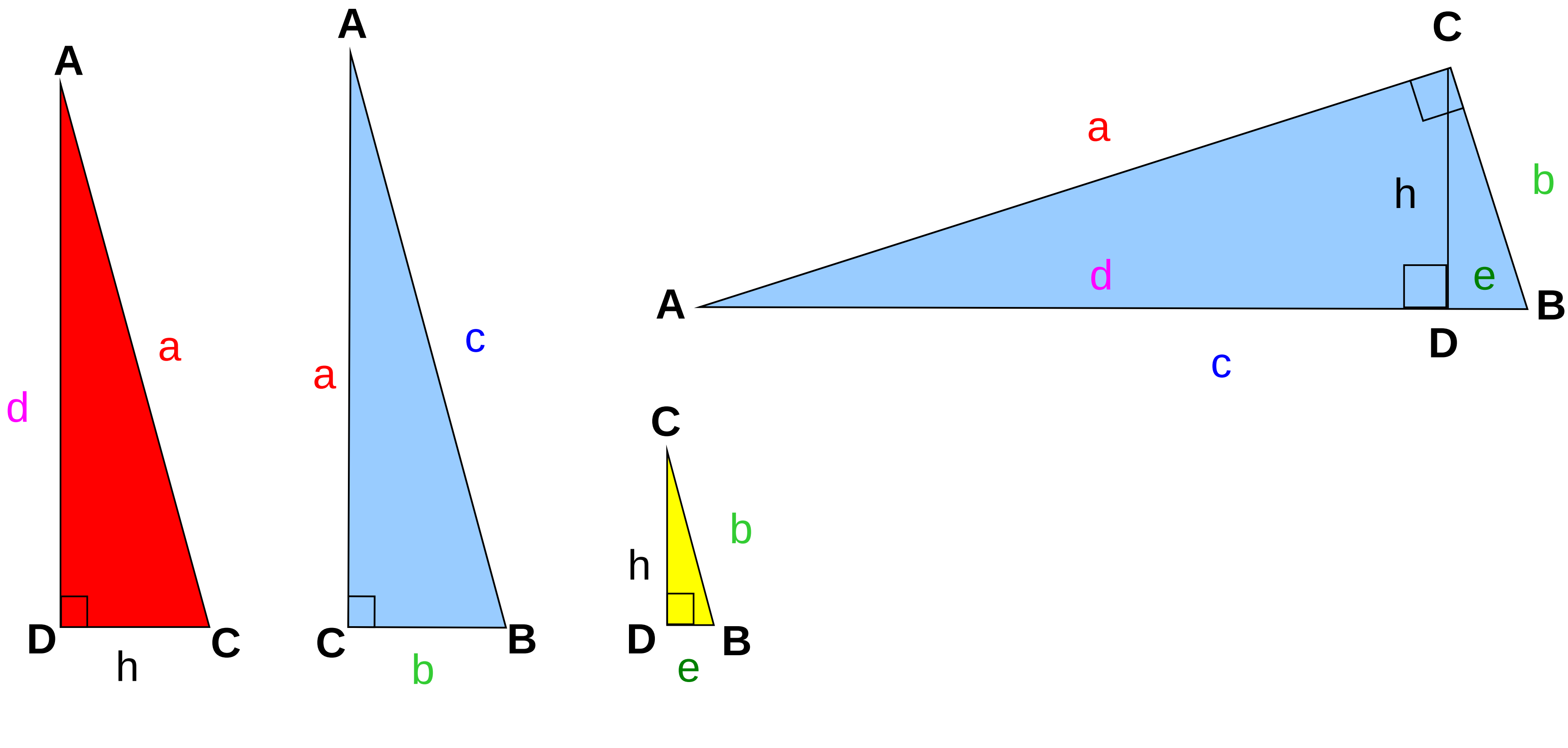
Geometric Mean

- We'll put the others up for reference



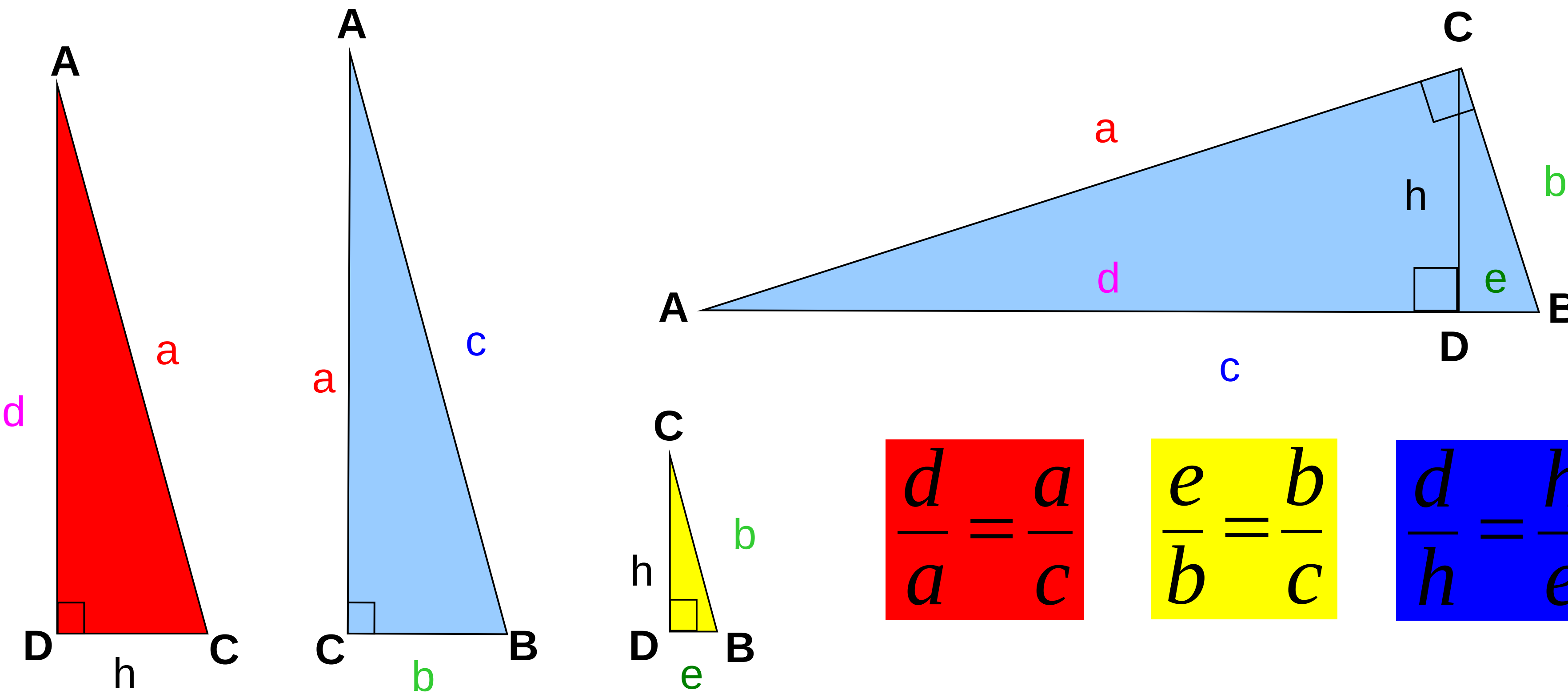
Geometric Mean

- Let's label the sides



Geometric Mean

- We can use similarity properties to set up proportions:



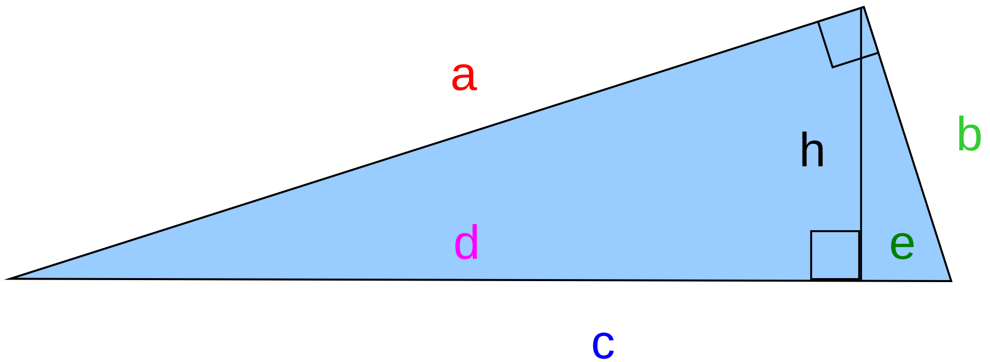
$$\frac{d}{a}=\frac{a}{c}$$

$$\frac{e}{b}=\frac{b}{c}$$

$$\frac{d}{h}=\frac{h}{e}$$

Geometric Mean

- To conclude, a, b, and h, can all be written as the Geometric Mean of two segments.



$$\frac{d}{a} = \frac{a}{c}$$

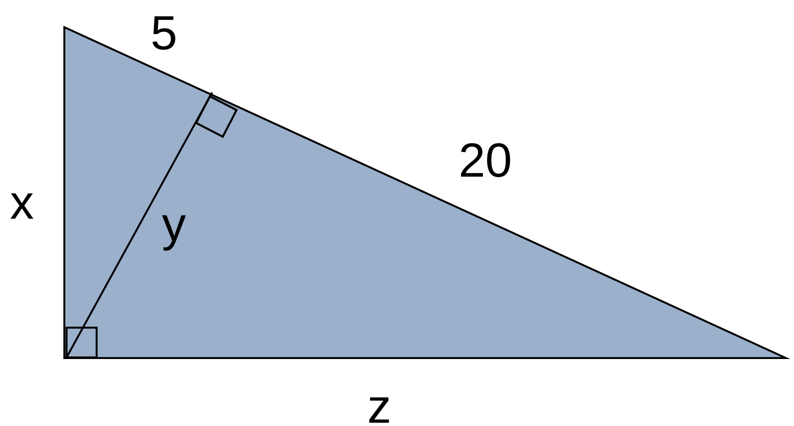
$$\frac{e}{b} = \frac{b}{c}$$

$$\frac{d}{h} = \frac{h}{e}$$

Geometric Mean

- Putting it in words:
- The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments.
  - The length of this altitude is the geometric mean between the two parts of the hypotenuse.
  - The length of a leg of this triangle is the geometric mean between the entire hypotenuse and the part of the hypotenuse next to that leg.

Example



$$\frac{25}{x} = \frac{x}{5}$$

$$\frac{5}{y} = \frac{y}{20}$$

$$\frac{25}{z} = \frac{z}{20}$$